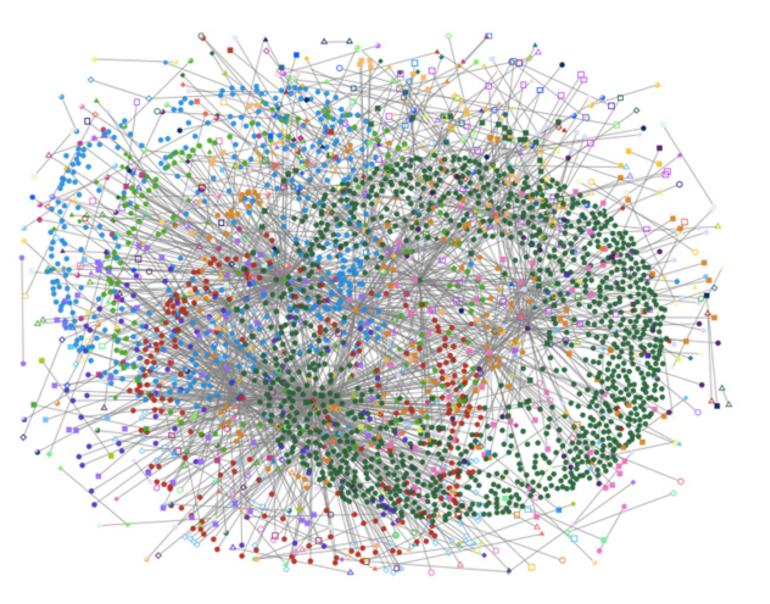


High-Performance Graph Analytics in Shared Memory

Hans Vandierendonck, Jiawen Sun

Queen's University Belfast

20 July 2018



WHAT ARE GRAPH ANALYTICS

Graphs represent interactions between people or things

Graph analytics are algorithms that extract information from a graph

Graphs tend to grow large, and often tend to exhibit a power-law degree distribution

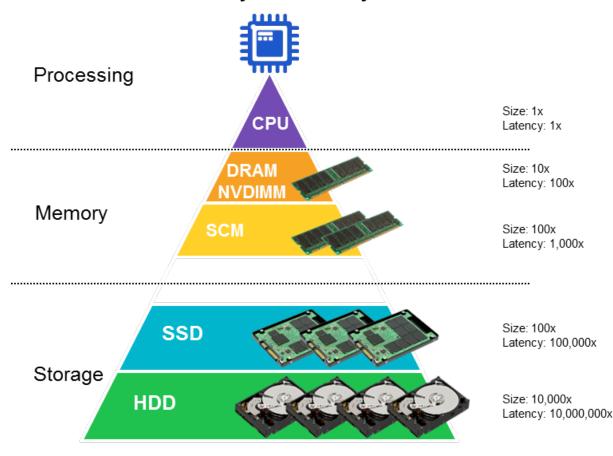
➢ "6 degrees of separation"



WHY SHARED MEMORY?

Because of properties of the workload

- Little computation, mostly communication/synchronisation
- Data sets not so large, e.g., Twitter's follower graph fits in memory of a single server [Sharma PVDLB'16]
- Future memory technologies will increase capacity: High-Bandwidth Memory/diestacking, storage-class memory
- Large-scale shared-memory systems implement a non-uniform memory access (NUMA) model



Memory Hierarchy

Image source: www.semiengineering.com

WHAT IS NUMA?

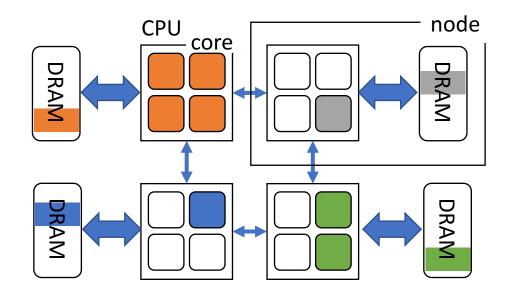
Each CPU socket is connected to local DRAM memory

Inter-node links provide access to "remote" DRAM memory

Local links have higher bandwidth and lower latency than inter-node links

Difference is more pronounced for stores than for loads

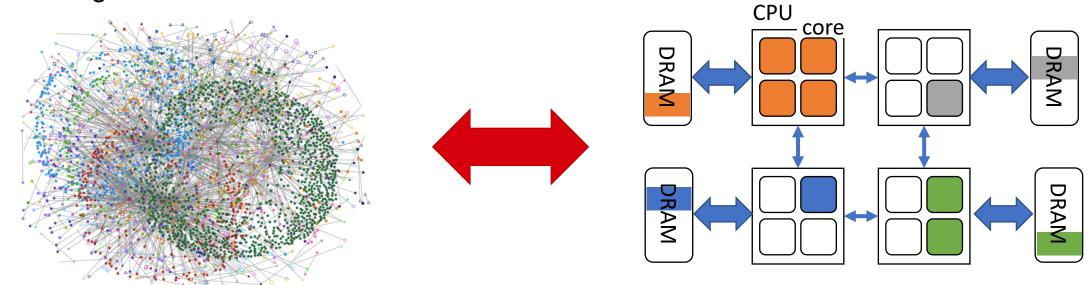
In a program optimised for NUMA, CPU cores primarily access local DRAM







How to map graph analytics over immutable graphs onto a NUMA architecture while minimising execution time?



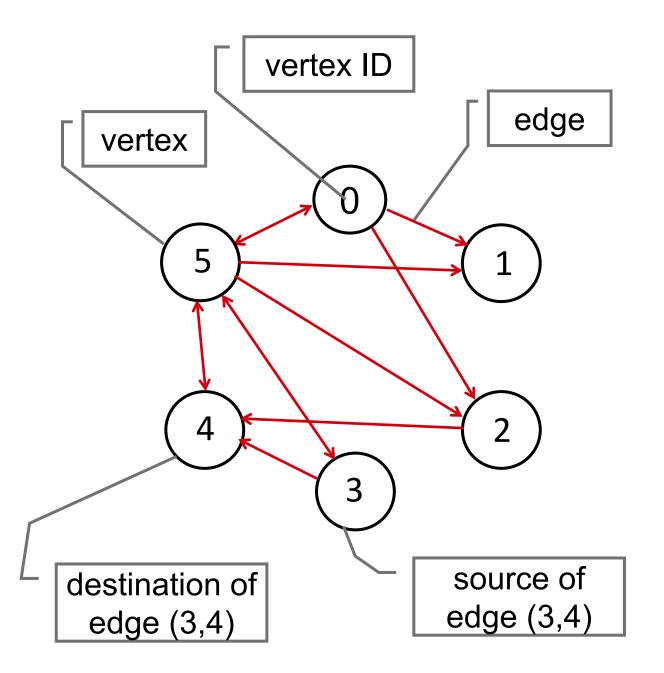


AGENDA

- Context and Goal
- Preliminaries
- Graph Algorithms
- Graph Analytics Frameworks
- Elements of High-Performance Graph Analytics
- NUMA-awareness
- Graph partitioning
- Load balance
- Conclusion and outlook



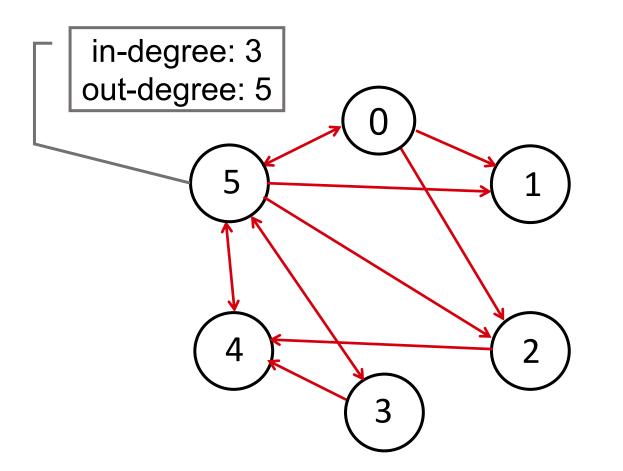




- Graph G=(V,E) where V: set of vertex labels
- $E \subseteq V \times V$: set of pairs of vertices

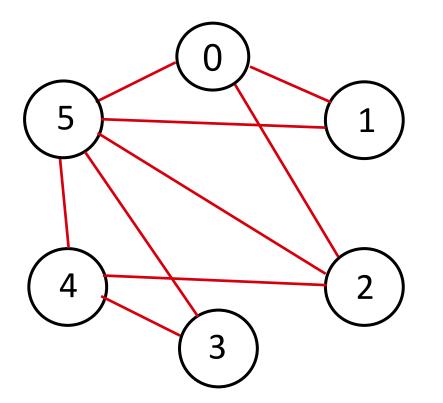
Frontier F is a set of active vertices with $\mathsf{F} \subseteq \mathsf{V}$





in-degree: #incoming edges
out-degree: #outgoing edges

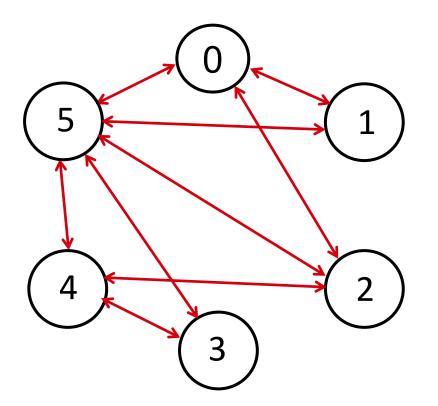




Directed graph: edges have a direction (source, destination)

Undirected graph: edges have no direction if (u,v) ϵ E then (v,u) ϵ E



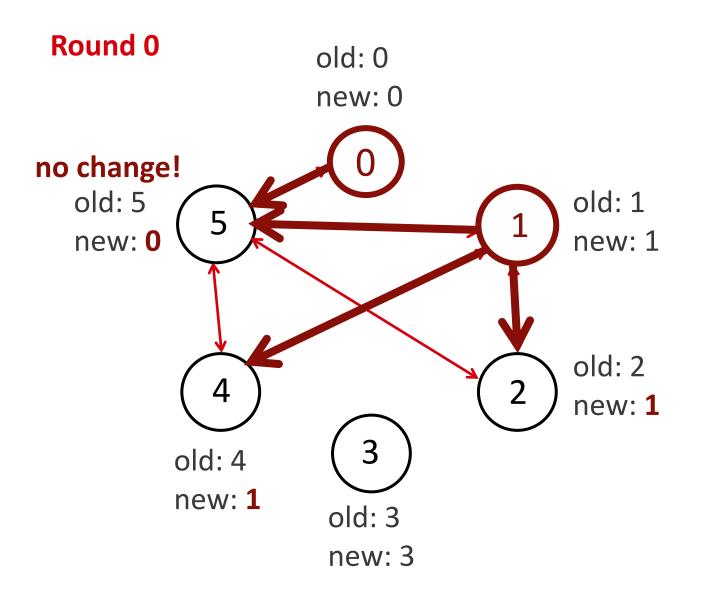


Undirected graphs are commonly represented such that every edge occurs twice



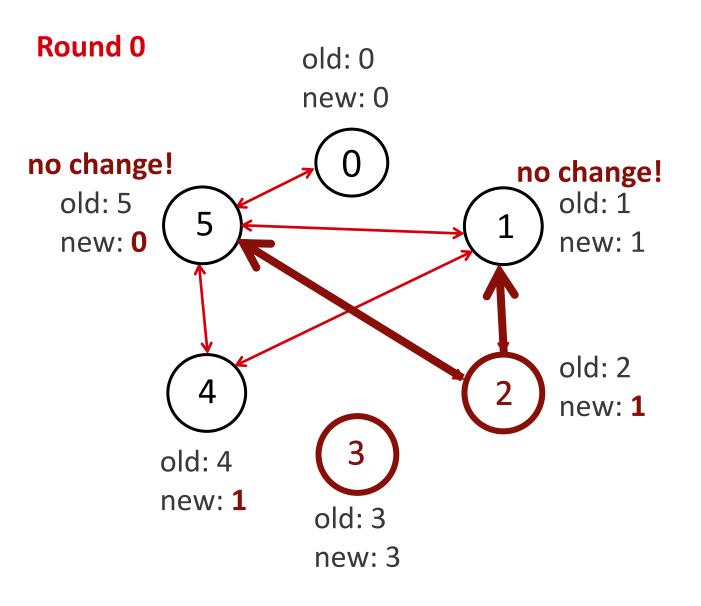
GRAPH ALGORITHMS





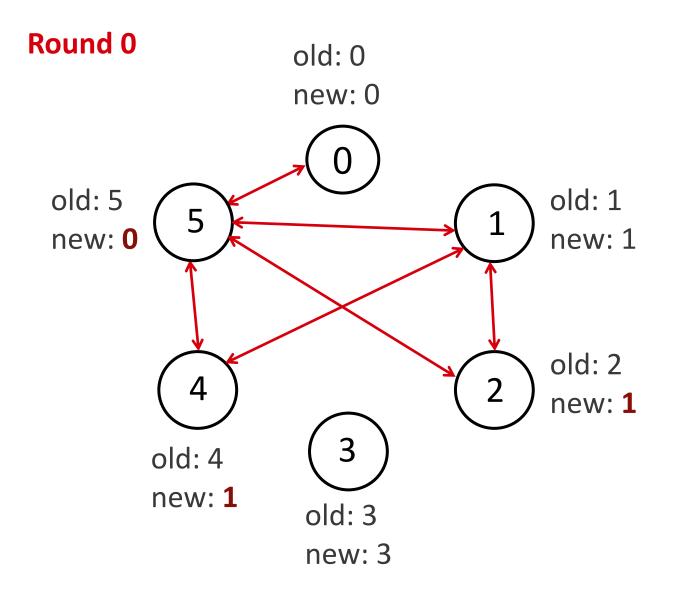
- Strongly connected components
- Initial label assignment of "old" label, copied to "new" label
- Update rule: for (u,v) in E:
 - new[v] = min(new[v],old[u])
- Copy "new" to "old" and repeat update phase until no more changes made





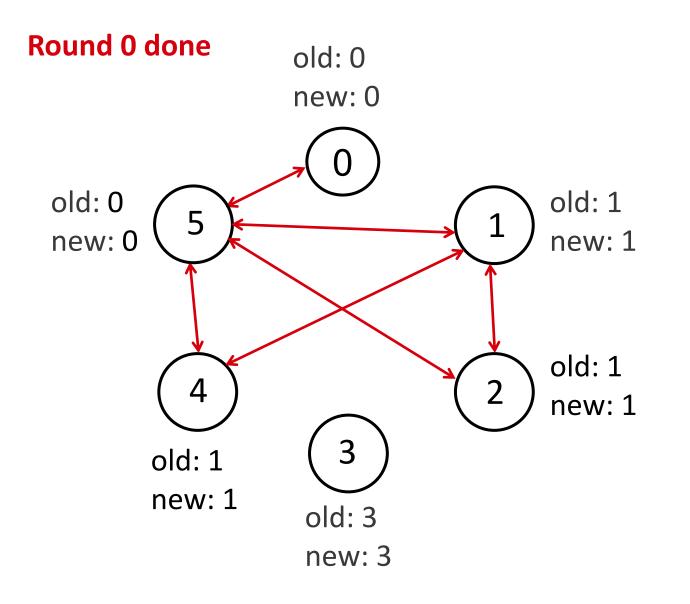
- Strongly connected components
- Initial label assignment of "old" label, copied to "new" label
- Update rule: for (u,v) in E:
 - new[v] = min(new[v],old[u])
- Copy "new" to "old" and repeat update rule on all edges until no more changes made





Further propagating the labels "4" and "5" held by vertices 4 and 5 incurs no changes in the labels of other vertices

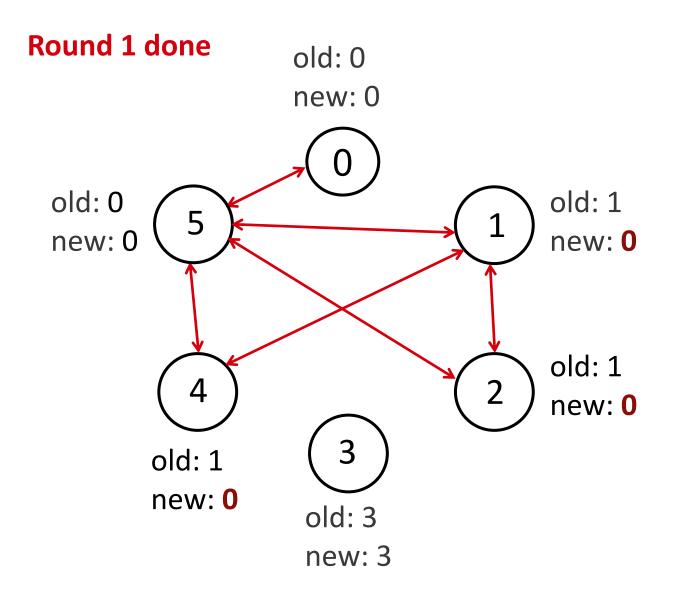




Round 0 of propagating labels has finished

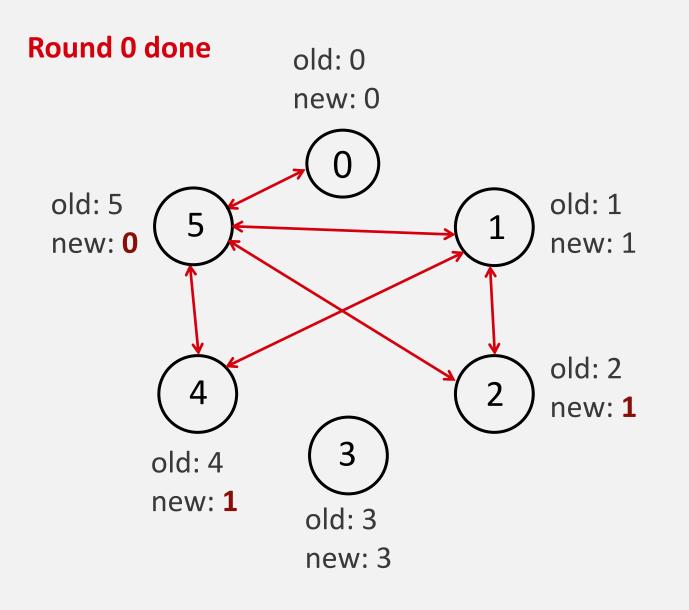
Copy "new" to "old" and go again...





After round 1, label "0" has propagated to more vertices We need to do one more round to ensure no further changes occur

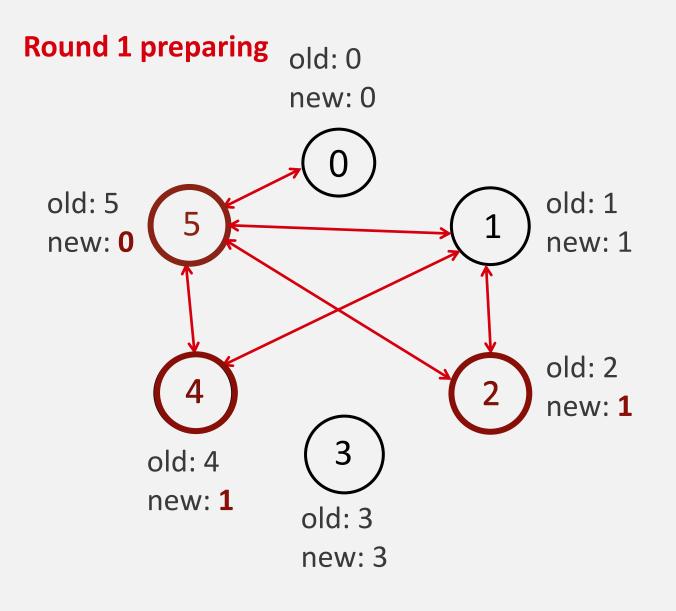




Let's return to the state at the end of round 0

If in any round the label did not change, then there is no point in trying to propagate the label again





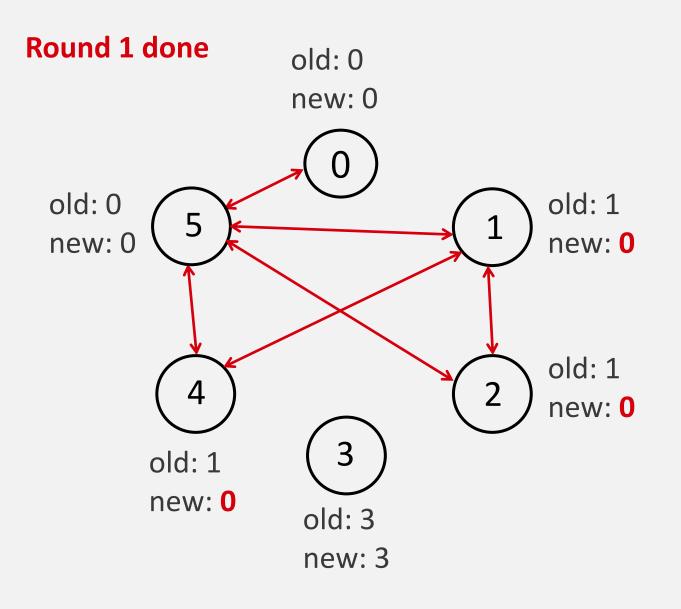
Let's return to the state at the end of round 0

Vertices 2, 4, 5 have changes
in their label (old[v]!=new[v])

Frontier = {2, 4, 5}

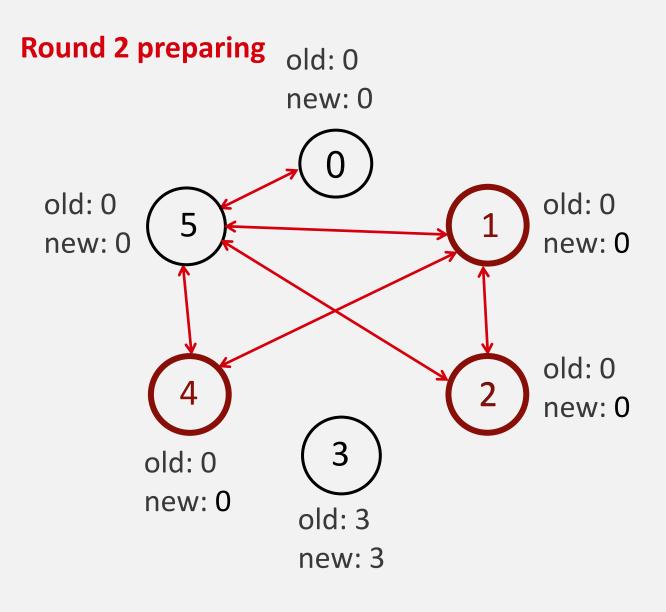
Only vertices 2, 4 and 5 are visited in round 1





Let's return to the state at the end of round 0 Vertices 2, 4, 5 have changes in their label (old[v]!=new[v]) Frontier = $\{2, 4, 5\}$ Only vertices 2, 4 and 5 are visited in round 1 In round 2: Frontier = $\{1, 2, 4\}$





Let's return to the state at the end of round 0 Vertices 2, 4, 5 have changes in their label (old[v]!=new[v]) Frontier = $\{2, 4, 5\}$ Only vertices 2, 4 and 5 are visited in round 1 In round 2: Frontier = $\{1, 2, 4\}$ Nothing changes

GRAPH ANALYTICS FRAMEWORKS



LIGRA

size(U : frontier) : N

returns |U|

EdgeMap(G : graph,

U : frontier,

F : (vertex × vertex) \rightarrow bool,

C : vertex \rightarrow bool) : frontier

VertexMap(U : frontier,

F : vertex \rightarrow bool) : frontier

[Shun PPoPP'13]

Assume graph G=(V,E)

EdgeMap applies an operation F to each edge $(u,v) \in E$ where $u \in U$ and C(v) = true. It returns a frontier that contains all v where any call to F(u,v) returned true

VertexMap applies an operation F to each vertex $v \in U$ and returns a frontier that contains v iff $v \in U$ and F(v) = true

In both cases, F may have side effects, e.g., updating properties for the vertices



Algorithm 8 Connected Components

1: $IDs = \{0, \dots, |V| - 1\}$ \triangleright initialized such that IDs[i] = i2: prevIDs = $\{0, \dots, |V| - 1\}$ \triangleright initialized such that prevIDs[i] = i3: 4: **procedure** CCUPDATE(s, d) $\operatorname{origID} = \operatorname{IDs}[d]$ 5: if (WRITEMIN(&IDs[d], IDs[s])) then 6: 7: **return** (origID == prevIDs[d]) 8: return 0 9: 10: procedure COPY(i) 11: prevIDs[i] = IDs[i]12: return 1 13: 14: procedure CC(G)Frontier = $\{0, ..., |V| - 1\}$ 15: \triangleright vertexSubset initialized to V while $(SIZE(Frontier) \neq 0)$ do 16: 17: Frontier = VERTEXMAP(Frontier, COPY) 18: Frontier = EDGEMAP(G, Frontier, CCUPDATE, C_{true}) 19: return IDs

Source: Shun PPoPP'13

LABEL PROPAGATION IN LIGRA

writeMin is an atomic "fetch_and_min" operation

Like compare-and-set, returns true if destination is successfully modified



```
interface GASVertexProgram(u) {
    // Run on gather_nbrs(u)
    gather(D_u, D_{(u,v)}, D_v) \rightarrow Accum
    sum(Accum left, Accum right) \rightarrow Accum
    apply(D_u, Accum) \rightarrow D_u^{new}
    // Run on scatter_nbrs(u)
    scatter(D_u^{new}, D_{(u,v)}, D_v) \rightarrow (D_{(u,v)}^{new}, Accum)
}
```

Figure 2: All PowerGraph programs must implement the stateless gather, sum, apply, and scatter functions.

Algorithm 1: Vertex-Program Execution Semantics

```
Input: Center vertex u
if cached accumulator a_u is empty then
foreach neighbor v in gather_nbrs(u) do
a_u \leftarrow sum(a_u, gather(D_u, D_{(u,v)}, D_v))
end
```

end

```
D_{u} \leftarrow \operatorname{apply}(D_{u}, a_{u})
foreach neighbor v scatter_nbrs(u) do
\begin{vmatrix} (D_{(u,v)}, \Delta a) \leftarrow \operatorname{scatter}(D_{u}, D_{(u,v)}, D_{v}) \\ \text{if } a_{v} \text{ and } \Delta a \text{ are not Empty then } a_{v} \leftarrow \operatorname{sum}(a_{v}, \Delta a) \\ \text{else } a_{v} \leftarrow \operatorname{Empty} \end{aligned}
end
```

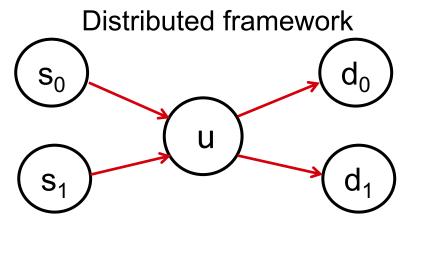
POWERGRAPH

[Gonzalez OSDI'12]

Similar concepts, presented differently

Vertices 'activated' by explicit call as opposed to recording frontier

Needs to maintain state on vertices and on edges





GIM-V, or 'Generalized Iterative Matrix-Vector multiplication' is a generalization of normal matrix-vector multiplication. Suppose we have a n by n matrix M and a vector vof size n. Let $m_{i,j}$ denote the (i, j)-th element of M. Then the usual matrix-vector multiplication is

 $M \times v = v'$ where $v'_i = \sum_{j=1}^n m_{i,j} v_j$.

There are three operations in the previous formula, which, if customized separately, will give a surprising number of useful graph mining algorithms:

- 1) combine2: multiply $m_{i,j}$ and v_j .
- combineAll: sum n multiplication results for node

 i.
- 3) assign: overwrite previous value of v_i with new result to make v'_i .

In GIM–V, let's define the operator \times_G , where the three operations can be defined arbitrarily. Formally, we have:

 $v' = M \times_G v$ where $v'_i = assign(v_i, combineAll_i(\{x_j \mid j = 1..n, and x_j = combine2(m_{i,j}, v_j)\})).$

PEGASUS

[Kang ICDM'09]

Similar concepts, presented differently

Uses connection between graphs and their adjacency matrix

Generalized matrix-vector multiplication captures 'accumulation' concept

Essentially says that graph algorithms may be represented as semi-rings



ELEMENTS OF HIGH-PERFORMANCE GRAPH ANALYTICS



GRAPH ANALYTICS STRUCTURE

```
frontier F := ...;
frontier newF := { };
for edge (u,v) \in E do
    if u \in F then
       if C(v) and op(u,v) then
          newF = newF \cup { v };
       fi
    fi
```

od

- **op** implements the update of vertex properties
- op, C are algorithm-specific
- op returns true if destination should be considered in the next round
- op(u,v) is usually of the form

new[v]=new[v]@old[v]
where @ is a commutative
and associative binary
operation (reduction)

• **C**(v) checks convergence

CONVERGENCE

od

```
frontier F := ...;
frontier newF := { };
for edge (u,v) \in E do
    if u \in F then
       if C(v) and op(u,v) then
          newF = newF \cup { v };
       fi
    fi
```

Shun PPoPP'13:

"The function C is useful in algorithms where a value associated with a vertex only needs to be updated once (i.e. breadth-first search)."

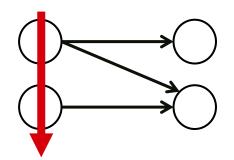
The paper also checks convergence for betweenness centrality

Real usefulness depends on how the graph is traversed

GRAPH DATA STRUCTURES

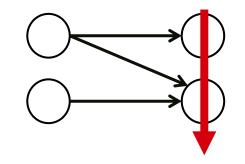
Compressed Sparse Rows (CSR)

- List outgoing edges for each vertex
- "Forward" traversal
- "Top-down" traversal
- "Push"
- "Vertex-centric"



Compressed Sparse Columns (CSC)

- List incoming edges for each vertex
- "Backward" traversal
- "Bottom-up" traversal
- "Pull"
- "Vertex-centric"



Frontier: CSR allows to skip edges for inactive vertices (u∉F)

Pruning: CSC allows to skip edges for pruned vertices (C(v)=false)

CSR-BASED EDGEMAP

frontier F := ...;
frontier newF := { };
for vertex u
$$\in$$
 V do
 if u \in F then
 for vertex v \in out(u) do
 if $\frac{C(v)}{v} = \frac{v}{v} \int \frac{v}{v}$;
find find

fi od fi od



- Checking frontier is compulsory: **op**(u,v) may be called only if u∈F
- Pruning is not compulsory: may call **op**(u,v) if **C**(v)=false
- As op(u,v) is only a handfull of instructions, there is little benefit in using C(v) in CSR

CSC-BASED EDGEMAP

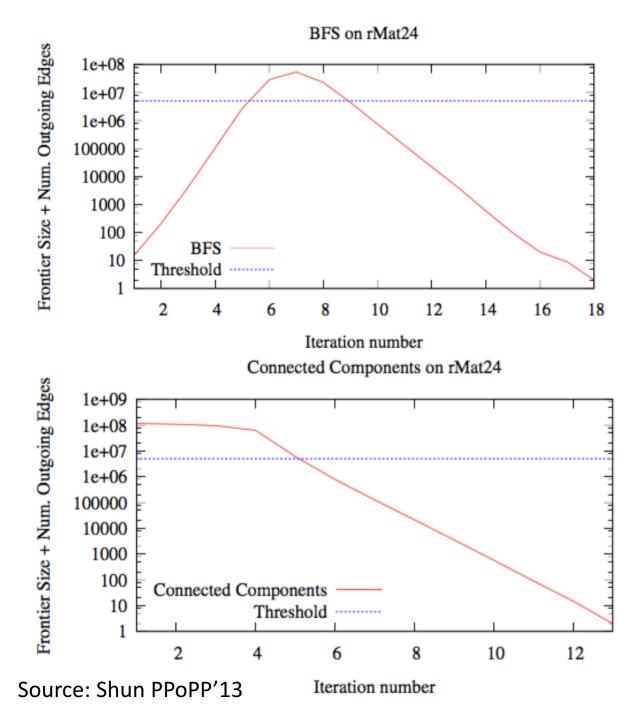
```
frontier F := ...;
frontier newF := { };
for vertex v \in V do
      if C(v) then
          for vertex u \in in(v) do
              if u \in F then
                     if op(u,v) then
                         newF = newF \cup \{v\};
                    if not C(v) then break; fi
fi fi fi od od
```

- Pruning is highly effective in CSC
- Allows to early terminate visiting the in-edges of u
- Or skip in-edges alltogether

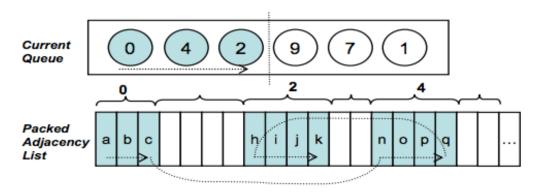
EVOLUTION OF FRONTIER SIZE

Algorithms exhibit one of three primary patterns:

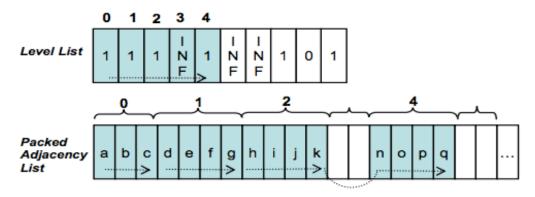
- flat
- shrink
- grow then shrink







(a) Data-Access Pattern of Queue-Based Method



(b) Data-Access Pattern of Read-Based Method

FRONTIER REPRESENTATION

"Sparse" frontiers

- Few bits set
- Queue of active vertex IDs

"Dense" frontiers

- Many bits set
- Bitmap or array of booleans

Dynamically switch as frontier size changes

[Hong et al, PACT'11]



CSR-BASED EDGEMAP WITH SPARSE FRONTIER

- frontier F := ...; // queue
- frontier newF := { };
- for vertex u \in F do

```
for vertex v \in out(u) do
```

if $\frac{\mathbf{c}(v)}{\mathbf{c}(v)}$ and op(u,v) then

```
newF = newF \cup { v }; // append
```

// queue

fi od od



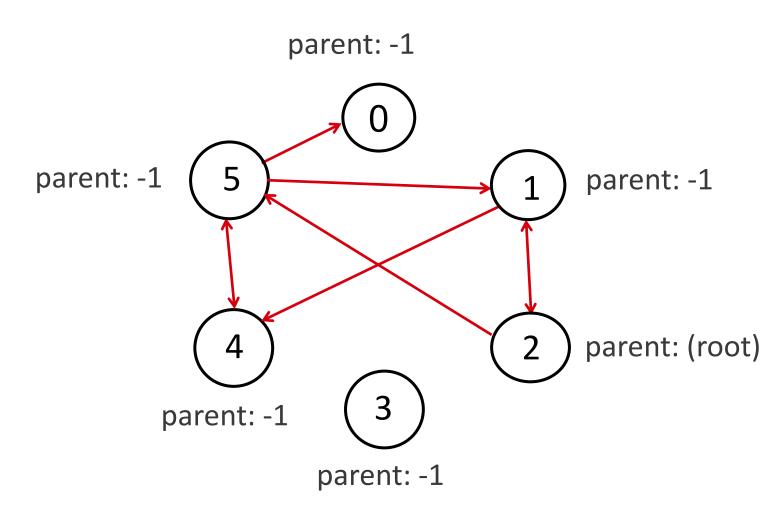
• Reminder: dense frontiers:

for vertex $u \in V$ do if $u \in F$ then

 Iteration over F is efficient when stored as a queue

...

- When F is stored as a queue, only CSR is efficient
- new frontier may contain duplicates!

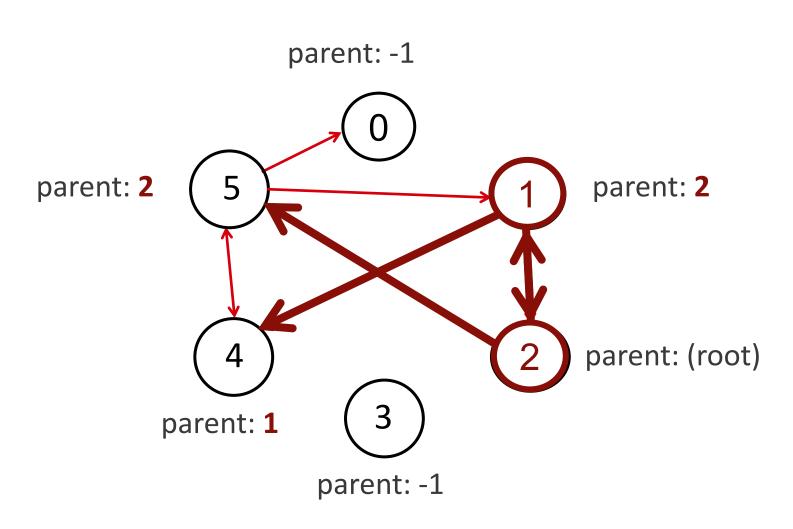


BREADTH-FIRST SEARCH

Starting from a root vertex, identify a shortest path to all other vertices Construct a spanning tree -1 means parent unknown In this case we start from vertex 2 Requires a frontier: all vertices that received a parent

in the previous round





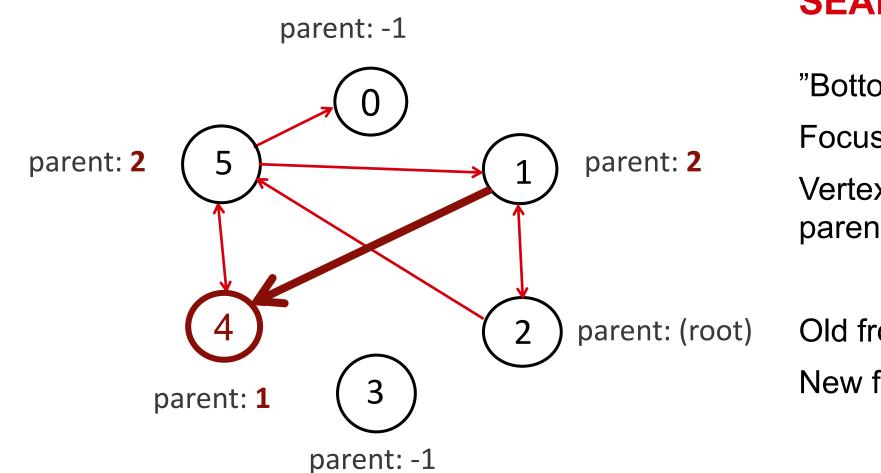
BREADTH-FIRST SEARCH

"Top-down" traversal, "push" Focus on out-edges

Old frontier: **1** 5

New frontier: 4 5





BREADTH-FIRST SEARCH

"Bottom-up" traversal, "pull" Focus on in-edges Vertex complete as soon as parent updated

ot) Old frontier: **1** 5 New frontier:



IMPACT OF CONVERGENCE [Beamer, SC12]

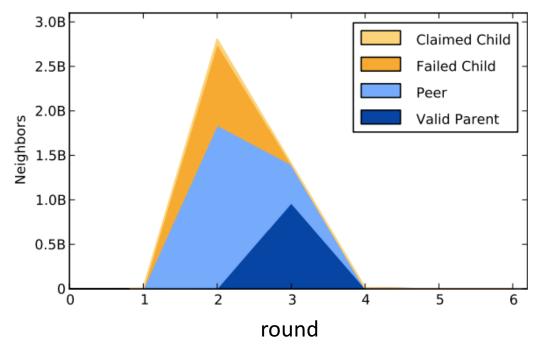


Fig. 3. Breakdown of edges in the frontier for a sample search on kron27 (Kronecker generated 128M vertices with 2B undirected edges) on the 16-core system.

Claimed child: parent[v] updated from -1 to a vertex ID Failed child: parent[v] updated in same round by parent Peer: parent[v] updated in same round by sibling Valid parent: v was encountered in a round prior to u

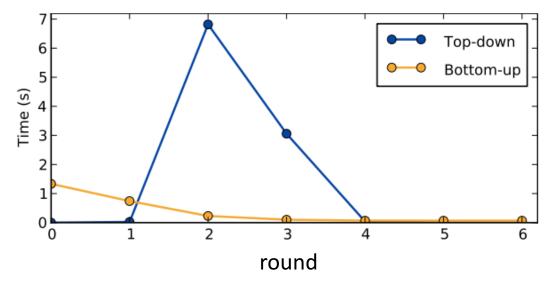


Fig. 6. Sample search on kron27 (Kronecker 128M vertices with 2B undirected edges) on the 16-core system.

Bottom-up/pull is faster in the middle rounds In those rounds, many vertices are active



DIRECTION-OPTIMISATION

[Beamer SC'12] [Shun SPAA'13]

d = (#active vertices +
#active edges) / #edges

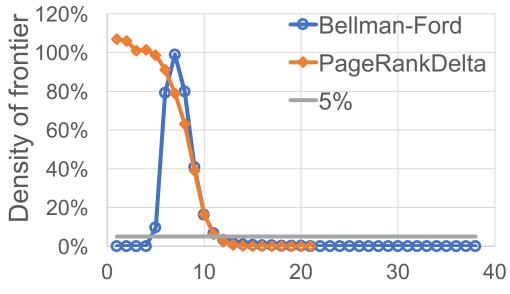
if d > 5% then
 # dense frontier
 if algorithm prefers
 forward then
 traverse CSR
else
 traverse CSC
endif
else # d <= 5%
 # sparse frontier
 traverse CSR
endif</pre>



Programmer's choice Little is known to guide this

We will shed some light on this ... work in progress

Requires storage of both CSC and CSR

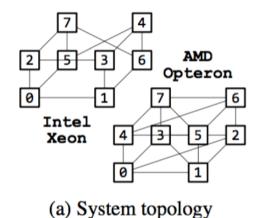


Iteration of algorithm

NUMA-AWARENESS



POLYMER



0-hop	1-hop	2-hop				
80-core Intel Xeon machine						
117	271	372				
108	304	409				
64-core AMD Opteron machine						
228	419	498				
256	463	544				
	re Intel 117 108 re AMD Op 228	re Intel Xeon ma 117 271 108 304 re AMD Opteron ma 228 419				

(b) Latencies (cycles) on the distance

Figure 3. The characteristics of NUMA machines for experiments.

Access	0-hop	1-hop	2-hop	Interleaved	
80-core Intel Xeon machine					
Sequential	3207	2455	2101	2333	
Random	720	348	307	344	
64-core AMD Opteron machine					
Sequential	3241	2806/2406	1997	2509	
Random	533	509/487	415	466	

Figure 4. The bandwidth (MB/s) of memory access on the distance.

[Zhang PPoPP'15]

Remote access has higher latency, lower bandwidth than local access

Stores are more affected than loads

Designed a scheme using graph partitioning [Kyrola OSDI'12] and privatization of vertex properties

We will discuss how their ideas were rehashed in GraphGrind



vertex property

EDGEMAP, VERTEXMAP AND NUMA-AWARENESS

Goal: map code and data to NUMA nodes

One type of arrays

• Properties (per vertex)

Two types of loops

- Loops over edges
- Loops over vertices

Two types of iteration

- Sparse frontier
- Dense frontier



RECAP: RACE CONDITIONS

A pair of load and store instructions, at least one of which is a store, that access the same memory location

In a concurrent program with race conditions, the outcome of the program may differ depending on the relative execution speed of threads Typical solutions:

- mutual exclusion
- atomic memory operations
- owner-computes



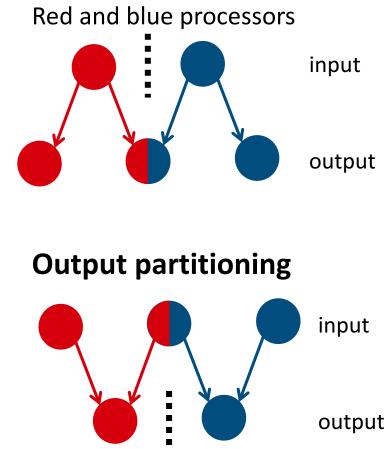
OWNER-COMPUTES

Decomposition based on partitioning input/output data is referred to as the owner computes rule

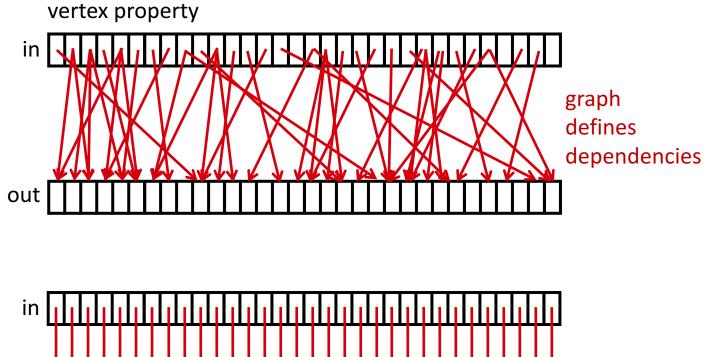
Each partition performs all the computations involving data that it owns

- Input data decomposition: A task performs all the computations that can be done using these input data
- **Output data decomposition:** A task computes all the results in the partition assigned to it

Input partitioning



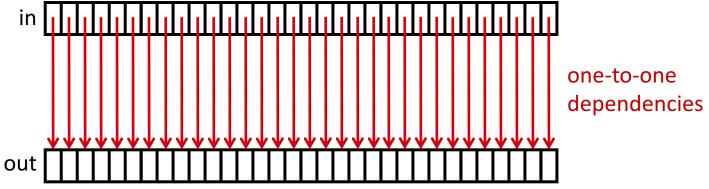




THE "MAP" IN EDGEMAP AND VERTEXMAP

Edgemap:

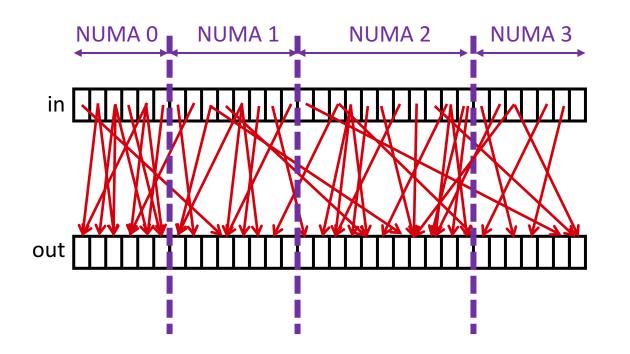
- Iteration space: (u,v) \in E where u, v \in V
- Dependencies are determined by graph topology



Vertexmap:

- Iteration space: $v \in V$





NUMA-AWARE LAYOUT FOR EDGEMAP

How to split edgemap over NUMA nodes?

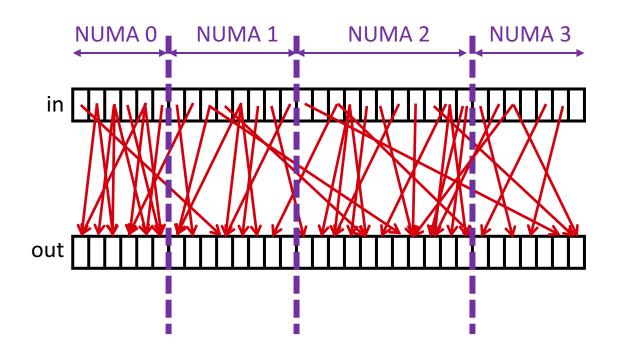
- code
- data

Observation

Remote stores are more expensive than remote loads [Zhang PPoPP '15]

Need to co-locate code with the updated data

Edges are processed by CPUs attached to the NUMA node that holds the destination's property



NUMA-AWARE LAYOUT FOR EDGEMAP

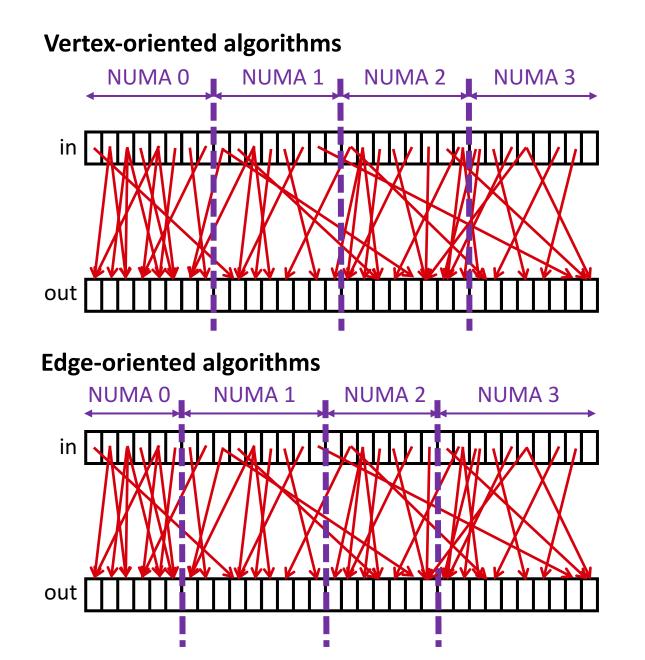
Goal

Determine cuts of { code, data } such that performance is maximised

How?

Partition graph such that each partition (NUMA node) has an equal:

- 1. #edges, #cuts [PowerGraph OSDI'12] ... breaks locality
- 2. #sources [X-stream SOSP'13] ... race conditions
- 3. #edges [Polymer PPoPP '15]
- 4. (α #destinations + #edges) [Gemini OSDI'16]



NUMA-AWARE LAYOUT FOR EDGEMAP

It depends! [GraphGrind ICS'17]

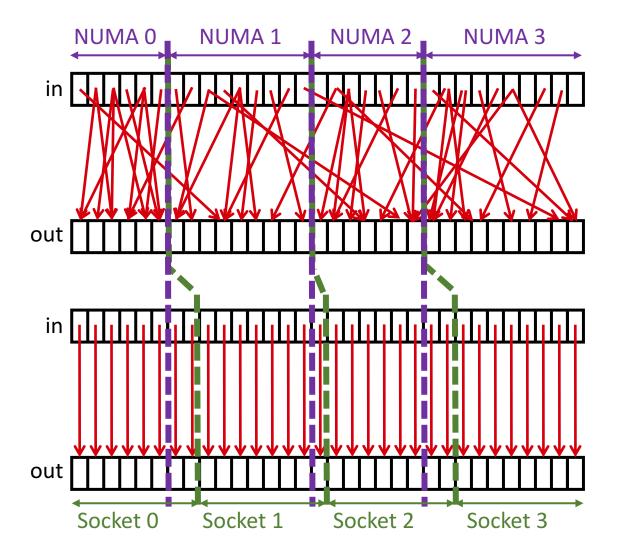
"Vertex-oriented" algorithms

- Best performance with equal #destinations
- Frontier density mostly below 50%
- BFS, Betweenness Centrality, Bellman-Ford

"Edge-oriented" algorithms

- Best performance with equal #edges
- Frontier density mostly close to 100%
- PageRank, SpMV, Belief Prop., PageRankDelta

Edge-oriented algorithms



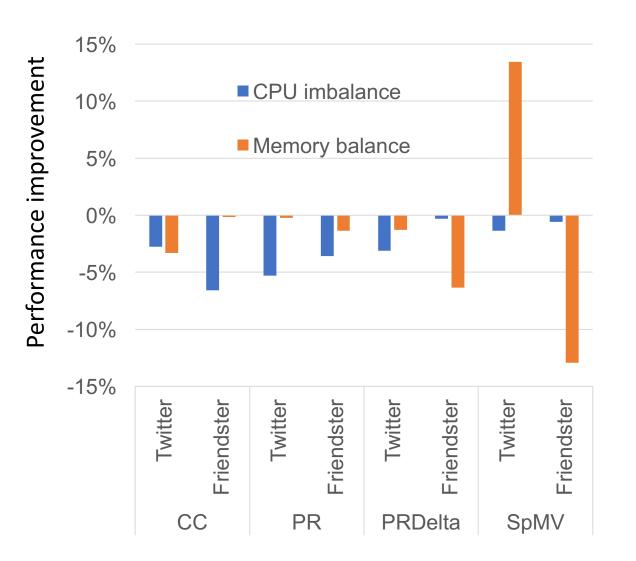
NUMA-AWARE LAYOUT FOR VERTEXMAP

"Vertex-oriented" algorithms

Trivial

"Edge-oriented" algorithms

- Need to choose between balancing compute and minimising traffic across NUMA nodes
- Better to balance compute and incur additional inter-node traffic [GraphGrind ICS'17]
- Consequently, data is partitioned differently from compute

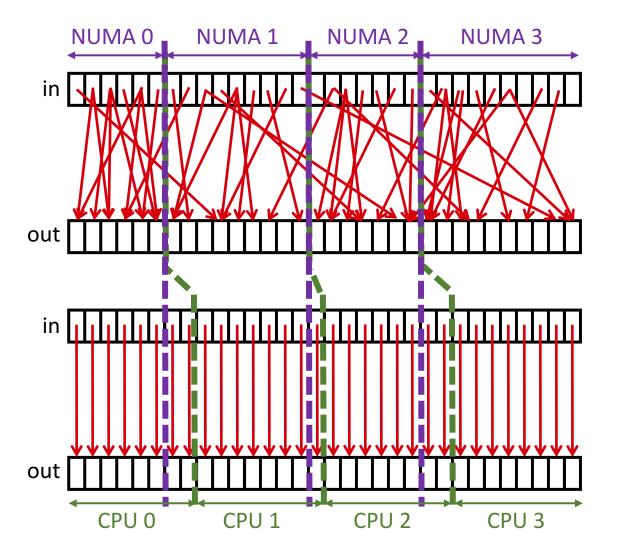


NUMA-AWARENESS CHOICES

- Baseline is CPU balance and memory imbalance
 - Implies remote accesses during vertex map
- CPU imbalance
 - No remote accesses during vertex map
- Memory balance
 - No remote accesses during vertex map
 - Many remote accesses during edge map



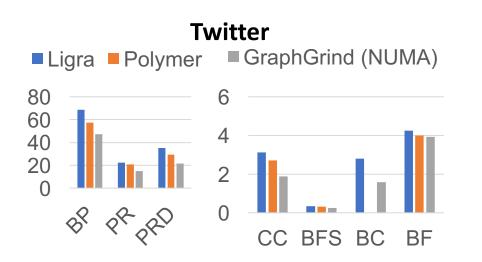
Edge-oriented algorithms

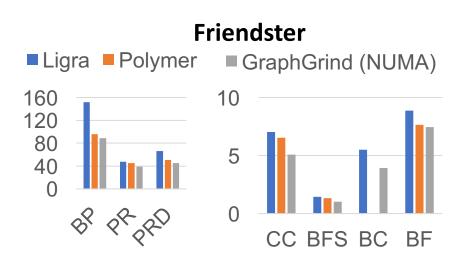


CAN WE MEET BOTH REQUIREMENTS?

Have our cake and eat it too!

VEBO





PERFORMANCE EVALUATION OF NUMA-AWARENESS

Combination of optimisations [Sun ICS'17]

- Pruned CSC/CSR representation
- Tune partitioning to edge/vertex algorithms
- NUMA-aware layout of vertex arrays
- CSC traversal: "caching" intermediate values to minimise load/stores
- Full frontier: specialised version of code that omits frontier check
- Sparse CSR traversal: no partitioning applied

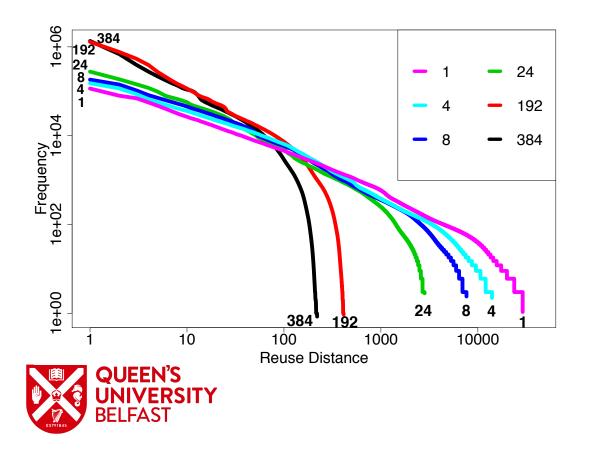
4-socket 2.6GHz Intel Xeon E7-4860 v2, 48 threads, 256 GiB

GRAPH PARTITIONING

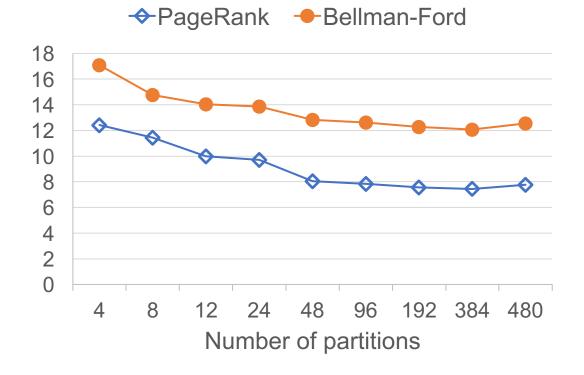


MEMORY LOCALITY

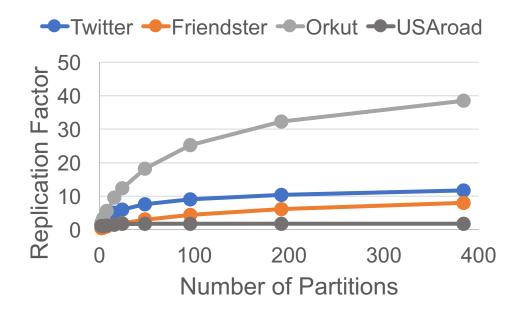
Reuse Distance Distribution



 Misses Per Kilo-Instruction (MPKI) – Twitter graph



VERTEX REPLICATION



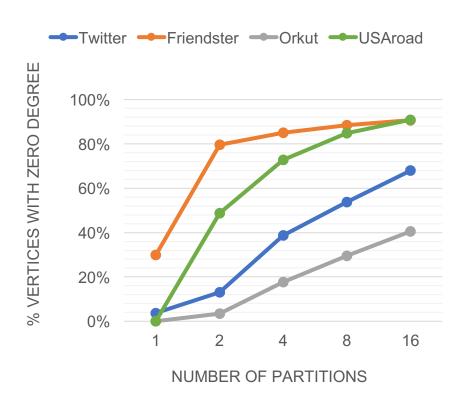
When partitioning the edge set, a vertex may appear in multiple partitions

Replication factor = #repeated vertices / #unique vertices

Replication factor tends to |E|/|V| as number of partitions grows

Replication implies space and runtime overhead





IMPLICATIONS OF VERTEX REPLICATION

A different view on the same effect:

If we partition the edge set P-way, then a vertex with degree d<P has zero edges in at least P-d partitions. It has some edges in at most d partitions.

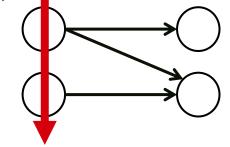
Partitions of a sparse graph are *hyper-sparse*



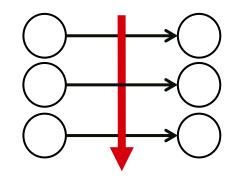
GRAPH DATA STRUCTURES

Compressed Sparse Rows (CSR)

- List outgoing edges for each vertex
- "Forward" traversal (push)
- "vertex-centric"



- Coordinate list (COO)
 - A list of edges
 - "edge-centric"

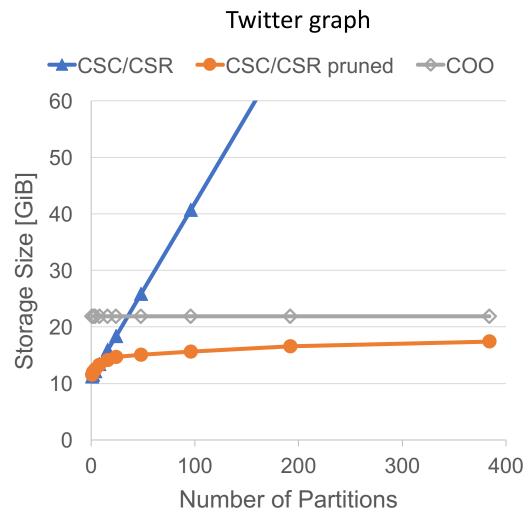


Compressed Sparse Columns (CSC)

- List incoming edges for each vertex
- "Backward" traversal (pull)
 - "vertex-centric"







IMPLICATIONS OF VERTEX REPLICATION

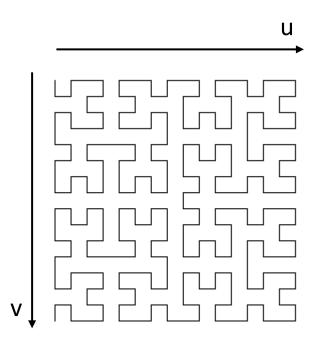
CSC and CSR are not scalable formats

- Increased storage
- Increased execution time to traverse graph

Pruned CSC/CSR

- A.k.a. Compressed Compressed
 Sparse Rows/Columns
- Omits zero-degree vertices
- COO is scalable to any number of partitions
 - But inefficient for sparse frontiers





Edges are points in a 2D space:

For u,v in 0,...,|V|-1: (u,v) = 1 if (u,v) \in E (u,v) = 0 otherwise Edgemap: visit all (u,v) in E

COO ADVANTAGE: SPACE FILLING CURVES

Space filling curves define a traversal order through a space that tends to minimise memory locality

Map *n*D order onto 1D order

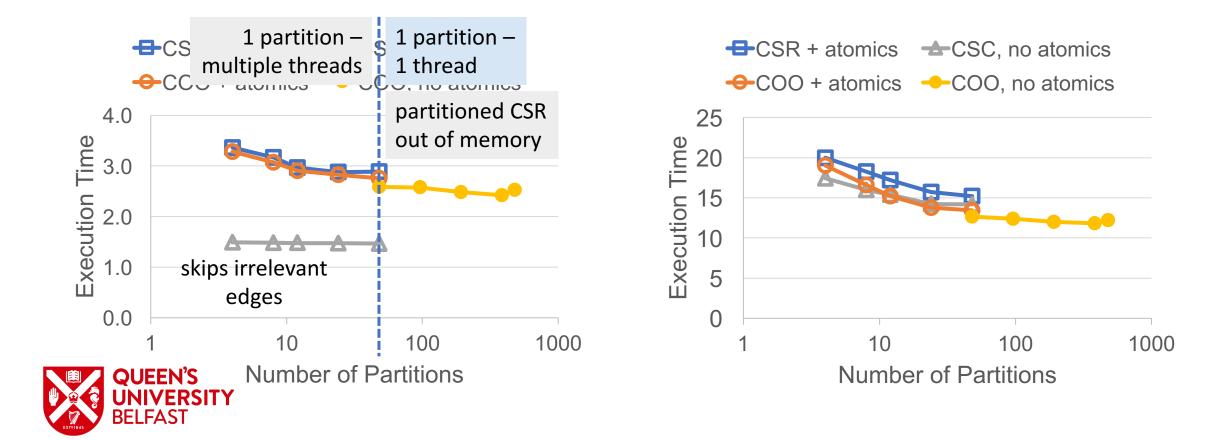
Hilbert curve, Morton order (Z-order), and many others

COO allows edges to be stored in any order:

- CSR order
- CSC order
- Space filling curves

GRAPH PARTITIONING BENEFITS

• Betweenness Centrality, Twitter



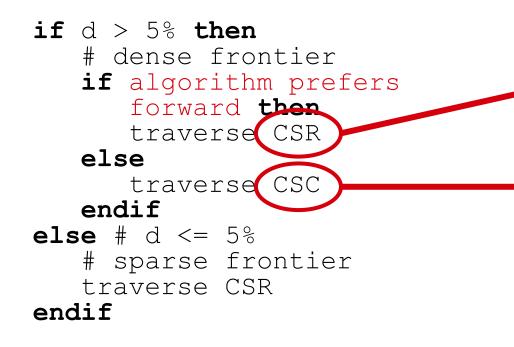
• PageRank, Twitter

10 iterations

DIRECTION-OPTIMIZATION

• Ligra [Shun PPoPP'13]

```
d = (#active vertices +
#active edges) / #edges
```



- GraphGrind [Sun ICPP'17]
- 3-way heuristic

d = (#active vertices +
#active edges) / #edges

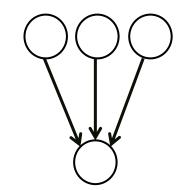
```
if d > 50% then
    # dense frontier
    traverse partitioned COO
else if d > 5% then
    # medium-dense case
    # dense frontier
    traverse CSC
else # d <= 5%
    # sparse frontier
    traverse CSR
endif</pre>
```

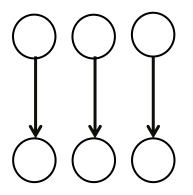


LOAD BALANCE



Revisting edge balance: Two partitions with 3 edges Which partition is processed faster?





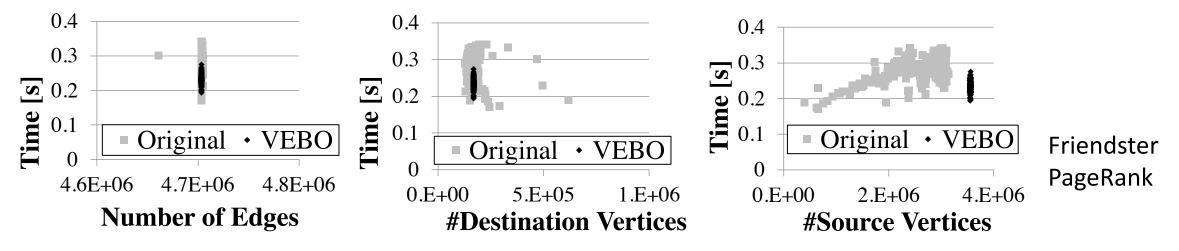
LOAD BALANCE

Execution time/partition highly dependent on the degree of vertices

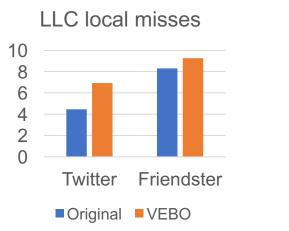
Reorder vertices

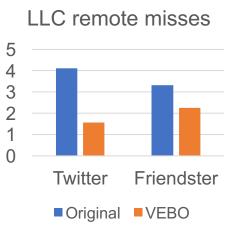
- in order of decreasing in-degree
- using list scheduling

VEBO: Vertex and Edge Balanced Partitioning



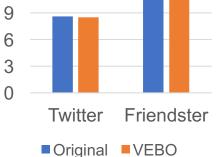
VEBO BENEFITS





LLC misses

12







■ Original ■ VEBO

Friendster

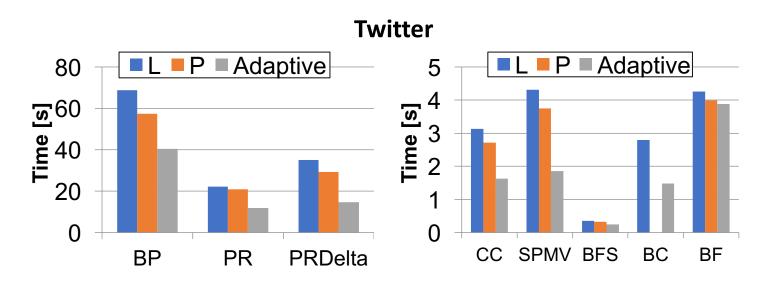
Twitter

Partitions are processed faster as a side-effect of reordering

Remote cache misses are traded for local misses

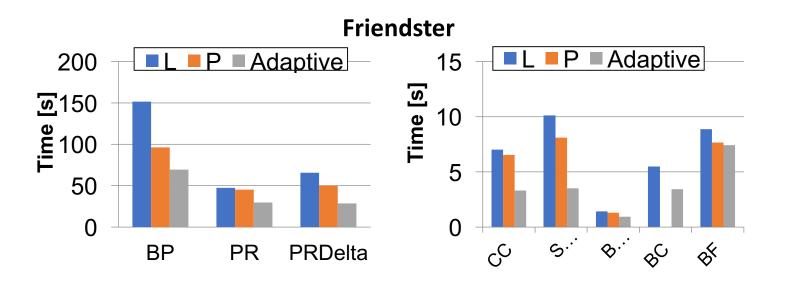


PageRank

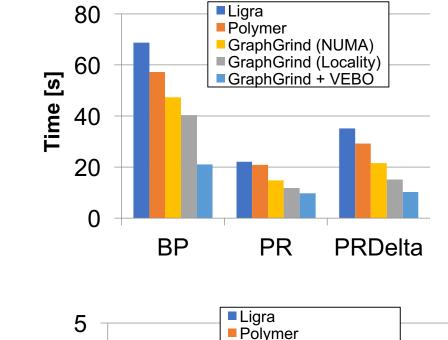


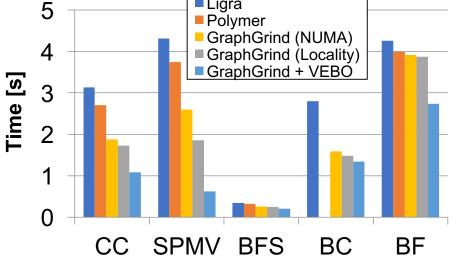
PERFORMANCE

L: Ligra, P: Polymer, Adaptive: GraphGrind with 3-way "directionoptimization"









PERFORMANCE

Comparing Ligra, Polymer (NUMA-aware), and 3 versions of GraphGrind

Twitter graph

4-socket 2.6GHz Intel Xeon E7-4860 v2, 48 threads, 256 GiB

Similar results hold for other graphs

VEBO relabels vertex IDs to achieve load balance



CONCLUSION AND OUTLOOK



CONCLUSION AND OUTLOOK

Scale-free properties of graphs make it hard to achieve high-performance

Code itself is short – devil is in the detail

Graph partitioning crucial: NUMA-locality; avoiding atomics; improving memory locality Some open questions:

- What are the limits on memory efficiency?
- What is the cause of performance difference between CSR/CSC/COO?
- Do the principles behind GraphGrind apply to distributed memory systems?
- How well does the programming model capture graph algorithms?



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